الامتحان الوطني الموحد للبكالوريا
المسالك الدولية – خيار أنجليزية
الدورة العادية 2018
-الموضوع-
NS 22E

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي
المركز الوطني للتقويم والامتحانات والتوجيه

3	مدة الإنجاز	الرياضيات	المادة
7	المعامل	مسلك علوم الحياة والأرض ومسلك العلوم الفيزيائية – خيار أنجليزية	الشعبة أو المسلك

GENERAL INSTRUCTIONS

- ✓ **The use of non- programmable calculator is allowed ;**
- ✓ **The exercises can be treated in the preferred order by the candidate ;**
- ✓ **The use of red color when writing solutions is to be avoided.**

COMPONENTS OF THE EXAM

- ✓ **The exam consists of three exercises and a problem , independent of each other according to the fields as follows :**

Exercise 1	**Geometry in space**	**3 points**
Exercise 2	**Complex numbers**	**3 points**
Exercise 3	**Calculating probabilities**	**3 points**
Problem	**Study of numerical function, calculating integrals and numerical sequences**	**11 points**

	Exercise 1 : (3 points)
	In the space referred to an orthonormal direct coordinate system $\left(O,\vec{i},\vec{j},\vec{k}\right)$,
	we consider the points $A(0,\ -2\ ,-2)$, $B(1,\ -2,-4)$ **and** $C(-3\ ,-1\ ,\ 2)$
1	**1) Show that** $\overrightarrow{AB}\wedge\overrightarrow{AC}=2\vec{i}+2\vec{j}+\vec{k}$ **and deduce that** $2x+2y+z+6=0$ **is a cartesian equation of the plane** (ABC)
	2) Let (S) **the sphere wich an equation is** $x^2+y^2+z^2-2x-2z-23=0$
0.5	**Verify that the sphere** (S) **has the center** $\Omega(1\ ,\ 0\ ,\ 1)$ **and the radius** $R=5$
0.25	**3) a) Verify that** $\begin{cases}x=1+2t\\y=2t\\z=1+t\end{cases}\ ;(t\in\mathbb{R})$ **is a parametric equations of the line** (Δ) **passing through the point** Ω **and perpendicular to the plane** (ABC)
0.5	**b) Determine the coordinates of** H **the point of intersection of the line** (Δ) **and the plane** (ABC)
0.75	**4) Verify that** $d\left(\Omega,(ABC)\right)=3$**, and then show that the plane** (ABC) **intersects the sphere** (S) **along a circle of radius** 4 **wich the center will be determined.**
	Exercise 2 : (3 points)
0.75	**1) Solve in the set of complex numbers** $\mathbb{C}$ **the equation** $2z^2+2z+5=0$
	2) In the complex plane referred to an orthonormal direct coordinate system $(O,\vec{u},\vec{v})$ **,**
	we consider the rotation R **with center** O **and angle** $\frac{2\pi}{3}$
0.25	**a) Write in trigonometric form the complex number** $d=\frac{-1}{2}+\frac{\sqrt{3}}{2}i$
0.5	**b) Let the point** A **of affix** $a=\frac{-1}{2}+\frac{3}{2}i$ **and the point** B **image of** A **by the rotation** R
	Let b **the affix of the point** B **, show that** $b=d.a$
	3) Let t **the translation with vector** $\overrightarrow{OA}$ **and the point** C **the image of** B **by** t **and** c **the affix of** C
0.75	**a) Verify that** $c=b+a$ **and deduce that** $c=a\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$ **(*you can use the question* 2) b))**
0.75	**b) Determine** $\arg\left(\frac{c}{a}\right)$ **and deduce that the triangle** OAC **is equilateral.**

Exercise 3 : (3 points)

An urn contains 9 balls, indistinguishable by touch : five red balls carrying the numbers

1 ; 1 ; 2 ; 2 ; 2 and four white balls carrying the numbers 1 ; 2 ; 2 ; 2

We consider the following experiment: we draw randomly and simultaneously three balls from the urn. We consider the events:

A : "The three balls drawn are of the same color"

B : " The three balls drawn carry the same number "

C : " The three balls drawn are of the same color and carry the same number "

1.5 — 1) Show that $p(A)=\frac{1}{6}$, $p(B)=\frac{1}{4}$ and $p(C)=\frac{1}{42}$

2) We repeat the previous experiment three times with returning the three balls drawn to the urn after each draw, and we consider X the random variable equal to the number of times of the realization of the event A.

0.5 — a) Determine the parameters of the binomial random variable X

1 — b) Show that $p(X=1)=\frac{25}{72}$ and calculate $p(X=2)$

Problem : (11 points)

I – We consider the numerical function g defined on IR by $g(x)=e^x-x^2+3x-1$

The table beside is the table of variations of the function g

x	$-\infty$ $\qquad$ $+\infty$
$g'(x)$	+
$g(x)$	$-\infty$ ↗ $+\infty$

0.25 — 1) Verify that $g(0)=0$

0.5 — 2) Determine the sign of $g(x)$ on each of the two intervals $]-\infty,0]$ and $[0,+\infty[$

II - We consider the numerical function f defined on IR by $f(x)=(x^2-x)\,e^{-x}+x$

and let (C) the curve of f in an orthonormal coordinate system $(O,\vec{i},\vec{j})$ (unit: $1cm$)

0.5 — 1) a) Verify that $f(x)=\frac{x^2}{e^x}-\frac{x}{e^x}+x$ for every x on IR and then show that $\lim\limits_{x\to+\infty} f(x)=+\infty$

0.75 — b) Calculate $\lim\limits_{x\to+\infty}(f(x)-x)$ and then deduce that (C) admits an asymptote (D) at $+\infty$ wich equation is $y=x$

0.5 — c) Verify that $f(x)=\frac{x^2-x+xe^x}{e^x}$ for every x on IR , and then calculate $\lim\limits_{x\to-\infty} f(x)$

0.5 — d) Show that $\lim\limits_{x\to-\infty}\frac{f(x)}{x}=-\infty$ and then interpret geometrically the obtained result

0.25	**2) a) Verify that** $f(x)-x$ **and** x^2-x **have the same sign for every** x **on** IR
0.5	**b) Deduce that** (C) **is above** (D) **on each of the intervals** $]-\infty,0]$ **and** $[1,+\infty[$ **,** **And below** (D) **on the interval** $[0,1]$
0.75	**3) a) Show that** $f'(x)=g(x)\,e^{-x}$ **for every** x **on** IR
0.5	**b) Deduce that the function** f **is decreasing on** $]-\infty,0]$ **and increasing on** $[0,+\infty[$
0.25	**c) Set up the table of variations of the function** f
0.25	**4) a) Verify that for every** x **on** IR **,** $f''(x)=(x^2-5x+4)e^{-x}$
0.5	**b) Deduce that the curve** (C) **admits two inflection points of respective abscissae 1 and 4**
1	**5) Sketch the line** (D) **and the curve** (C) **in the same system coordinate** $(O,\vec{i},\vec{j})$ **(we take** $f(4)\approx 4,2$ **)**
0.5	**6) a) Show that the function** $H:x\mapsto(x^2+2x+2)e^{-x}$ **is a primitive of the function** $h:x\mapsto -x^2e^{-x}$ **on** IR **, and then deduce that** $\int_0^1 x^2e^{-x}dx=\frac{2e-5}{e}$
0.75	**b) Using an integration by parts, show that** $\int_0^1 xe^{-x}dx=\frac{e-2}{e}$
0.75	**c) Calculate, in** cm^2**, the area enclosed between the curve** (C) **, the line** (D)**, and the lines of equations** $x=0$ **and** $x=1$
	III- We consider the numerical sequence (u_n) **defined by** $u_0=\frac{1}{2}$ **and** $u_{n+1}=f(u_n)$ **for every natural number** n
0.75	**1) Show that** $0\le u_n\le 1$ **for every natural number** n **(***you can use the result of the question* II-3)b) **)**
0.5	**2) Show that the sequence** (u_n) **is decreasing .**
0.75	**3) Deduce that** (u_n) **is convergent and determine its limit.**