المادة	الرياضيات	مدة الإنجاز	3
الشعبة أو المسلك	شعبة العلوم التجريبية مسلك علوم الحياة والأرض ومسلك العلوم الفيزيائية (خيار إنجليزية)	المعامل	7

GENERAL INSTRUCTIONS

- ✓ **The use of non- programmable calculator is allowed ;**
- ✓ **The exercises can be treated in the preferred order by the candidate ;**
- ✓ **The use of red color when writing solutions is to be avoided.**

COMPONENTS OF THE EXAM

The exam consists of three exercises and a problem , independent of each other according to the fields as follows :

Exercise 1	**numerical sequences**	**4 points**
Exercise 2	**Complex numbers**	**5 points**
Exercise 3	**Limits, differentiability and calculating integrals**	**4 points**
Problem	**Study of numerical function**	**7 points**

- ✓ $\overline{z}$ **denotes the conjugate of the complex number** z
- ✓ **ln denotes the Napierian logarithm function**

	Exercise 1 : (4 points)
	Let (u_n) **be the numerical sequence defined by** $u_0=\frac{3}{2}$ **and** $u_{n+1}=\frac{2u_n}{2u_n+5}$ **for every natural number** n
0.25	**1) calculate** u_1
0.5	**2) Show by induction that** $u_n>0$ **for every natural number** n
1	**3) a) Show that:** $0<u_{n+1}\le\frac{2}{5}u_n$ **; for every natural number** n **then deduce that ,** $0<u_n\le\frac{3}{2}\left(\frac{2}{5}\right)^n$ **for every natural number** n **.**
0.5	**b) Calculate** $\lim u_n$
	4) we consider the numerical sequence (v_n) **defined by** $v_n=\frac{4u_n}{2u_n+3}$ **for every natural number** n
0.75	**a) Show that** (v_n) **is a geometrical sequence of reason** $\frac{2}{5}$
1	**b) Determine** v_n **in terms of** n **therefore deduce** u_n **in terms of** n **for every natural number** n**.**
	Exercise 2 : (5 points)
	1) In the set of complex numbers $\mathbb{C}$ **we consider the equation** $(E):z^2-2\left(\sqrt{2}+\sqrt{6}\right)z+16=0$
0.5	**a) Verify that the discriminant of the equation** (E) **is** $\Delta=-4\left(\sqrt{6}-\sqrt{2}\right)^2$
1	**b) Deduce the solutions of equation** (E) **.**
	2) Let the complex numbers $a=\left(\sqrt{6}+\sqrt{2}\right)+i\left(\sqrt{6}-\sqrt{2}\right)$, $b=1+i\sqrt{3}$ **and** $c=\sqrt{2}+i\sqrt{2}$
0.75	**a) Verify that** $b\overline{c}=a$ **and deduce that** $ac=4b$
0.5	**b) Write the complex numbers** b **and** c **in trigonometric form.**
0.5	**c) Deduce that** $a=4\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$
	3) In the complex plane referred to an orthonormal direct coordinate system $(O,\vec{u};\vec{v})$ **,** **we consider the points** B, C **and** D **of respective affixes** b, c **and** d **such that** $d=a^4$ **Let** z **be the affix of a point** M **in the complex plane and** z' **the affix of the point** M' **image of** M **by the rotation** R **with center** O **and angle** $\frac{\pi}{12}$
0.5	**a) Verify that** $z'=\frac{1}{4}az$
0.25	**b) Determine the image of the point** C **by the rotation** R
0.25	**c) Determine the nature of the triangle** OBC **.**
0.75	**d) Show that** $a^4=128b$ **and deduce that the points** O,B **and** D **are collinear.**

Exercise 3 : (4 points)

Consider the numerical function g **defined on** $]0;+\infty[$ **by** $g(x)=2\sqrt{x}-2-\ln x$

0.5 **1) a) Show that for every** x **in** $]0;+\infty[$ **;** $g'(x)=\dfrac{\sqrt{x}-1}{x}$

0.5 **b) Show that** g **is increasing on** $[1;+\infty[$ **;**

0.5 **c) Deduce that for every** x **in** $[1;+\infty[$ **;** $0\leq \ln x\leq 2\sqrt{x}$ **; (Notice that** $2\sqrt{x}-2\leq 2\sqrt{x}$ **)**

1 **d) Show that for every** x **in** $[1;+\infty[$ **:** $0\leq \dfrac{(\ln x)^3}{x^2}\leq \dfrac{8}{\sqrt{x}}$ **therefore deduce** $\lim\limits_{x\to+\infty}\dfrac{(\ln x)^3}{x^2}$ **;**

0.75 **2) a) Show that the function** G **defined by** $G(x)=x\left(-1+\dfrac{4}{3}\sqrt{x}-\ln x\right)$ **is a primitive of the function** g **on** $]0;+\infty[$ **.**

0.75 **b) Calculate the integral** $\int_1^4 g(x)dx$ **.**

Problem : (7 points)

Consider the numerical function f **defined on** $\mathbb{R}$ **by** $f(x)=-x+\dfrac{5}{2}-\dfrac{1}{2}e^{x-2}\left(e^{x-2}-4\right)$

and (C) **the curve of** f **in an orthonormal coordinate system** $(O,\vec{i};\vec{j})$ **(unit:** $2cm$ **)**

0.5 **1) Show that** $\lim\limits_{x\to-\infty} f(x)=+\infty$ **and** $\lim\limits_{x\to+\infty} f(x)=-\infty$

0.5 **2) a) Show that the line** (Δ) **of equation** $y=-x+\dfrac{5}{2}$ **is an asymptote to the curve** (C) **near** $-\infty$ **.**

0.75 **b) Solve the equation** $e^{x-2}-4=0$**, therefore show that the curve** (C) **is above** (Δ) **on the interval** $]-\infty,2+\ln 4]$ **and below** (Δ) **on the interval** $[2+\ln 4,+\infty[$

0.5 **3) Show that** $\lim\limits_{x\to+\infty}\dfrac{f(x)}{x}=-\infty$ **and interpret geometrically the obtained result**

0.5 **4) a) Show that for every** x **in** $\mathbb{R}$ **:** $f'(x)=-\left(e^{x-2}-1\right)^2$

0.25 **b) Set up the table of variations of the function** f

0.75 **5) Calculate** $f''(x)$ **for every** x **in** $\mathbb{R}$ **therefore show that** $A(2,2)$ **is an inflection point of the curve** (C)

0.5 **6) Show that the equation** $f(x)=0$ **admits an unique solution** α **such that** $2+\ln 3<\alpha<2+\ln 4$

1 **7) Sketch the line** (Δ) **and the curve** (C) **in the same coordinate system** $(O,\vec{i};\vec{j})$

(Take $\ln 2\simeq 0,7$ **and** $\ln 3\simeq 1,1$**)**

0.5	**8)a) Show that the function** f **admits an inverse function** f^{-1} **defined on** $\mathbb{R}$ **.**
0.75	**b) Sketch in the same coordinate system** $\left(O,\vec{i};\vec{j}\right)$ **the curve of the function** f^{-1} **(Notice that the line** (Δ) **is perpendicular to the first bisector of coordinate system)**
0.5	**c) Calculate** $\left(f^{-1}\right)'(2-\ln 3)$ **(Notice that** $f^{-1}(2-\ln 3)=2+\ln 3$ **)**