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المسالك الدولية - خيار إنجليزية
الدورة العادية 2017
- الموضوع -

NS 22E

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي

المركز الوطني للتقويم والامتحانات والتوجيه

3	مدة الإنجاز	الرياضيات	المادة
7	المعامل	مسلك العلوم الفيزيائية – خيار إنجليزية	الشعبة أو المسلك

GENERAL INSTRUCTIONS

- The use of non- programmable calculator is allowed ;
- The exercises can be treated in the preferred order by the candidate ;
- The use of red color when drafting solutions is to be avoided.

COMPONENTS OF THE EXAM

- The exam consists of three exercises and a problem , independent of each other according to the fields as follows :

Exercise 1	Geometry in space.	3 points
Exercise 2	Calculating probabilities.	3 points
Exercise 3	Complex numbers.	3 points
Problem	Study of numerical function, calculating integrals and numerical sequences.	11 points

- Concerning the problem , ln denotes the Napierian logarithm function .

Exercise 1 (3 points)

In the space referred to an orthonormal direct coordinate system $\left(O,\vec{i},\vec{j},\vec{k}\right)$,

we consider the plane (P) passing through the point $A(0,1,1)$ and having $\vec{u}\,(1,0,-1)$ as

a normal vector and the sphere (S) with the center $\Omega(0,1,-1)$ and the radius $\sqrt{2}$

0.5 1) a) Show that $x-z+1=0$ is a cartesian equation of the plane (P)

0.75 b) Show that the plane (P) is tangent to the sphere (S) and verify that the plane (P)

Intersects the sphere (S) at the point $B(-1,1,0)$

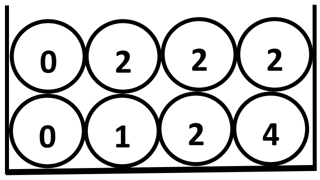
0.25 2) a) Determine a parametric equations of the line (Δ) passing through the point A and

perpendicular to the plane (P)

0.75 b) Show that the line (Δ) is tangent to the sphere (S) at the point $C(1,1,0)$

0.75 3) Show that $\overrightarrow{OC}\wedge\overrightarrow{OB}=2\vec{k}$ and then deduce the area of the triangle OCB

Exercise 2 (3 points)



An urn contains eight balls , indistinguishable by touch,

each carrying a number , as shown in the figure beside .

We draw , simultaneously and randomly , three balls from the urn.

1.5 1) Let A be the event: « Among the three drawn balls , no ball carries the number 0 »

and B the event:«The product of the numbers carried by the three drawn balls is equal to 8 »

Show that $p(A)=\dfrac{5}{14}$ and that $p(B)=\dfrac{1}{7}$

2) Let X be the random variable that associates to each draw the product of the numbers

carried by the three drawn balls.

0.5 a) Show that $p(X=16)=\dfrac{3}{28}$

1 b) The table beside concerns the law of the probability

of the random variable X

x_i	0	4	8	16
$p(X=x_i)$				$\dfrac{3}{28}$

Copy the table on your copy and complete it by justifying each answer.

Exercise 3 (3 points)

We consider the complex numbers a and b such that $a=\sqrt{3}+i$ and $b=\sqrt{3}-1+\left(\sqrt{3}+1\right)i$

0.25 1) a) Verify that $b=(1+i)a$

0.5 b) Deduce that $|b|=2\sqrt{2}$ and that $\arg b\equiv\frac{5\pi}{12}\ [2\pi]$

0.5 c) Deduce from the previous that $\cos\frac{5\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}$

2) The complex plane is referred to an orthonormal direct coordinate system $(O,\vec{u},\vec{v})$

We consider the points A and B of respective affixes a and b and the point C of affix c such that $c=-1+i\sqrt{3}$

0.75 a) Verify that $c=ia$ and then deduce that $OA=OC$ and that $\left(\overrightarrow{OA},\overrightarrow{OC}\right)\equiv\frac{\pi}{2}\ [2\pi]$

0.5 b) Show that the point B is the image of the point A by the translation with vector $\overrightarrow{OC}$

0.5 c) Deduce that the quadrilateral $OABC$ is a square.

Problem (11 points)

I- Let g be the numerical function defined on the interval $]0,+\infty[$ by: $g(x)=x^2+x-2+2\ln x$

0.25 1) Verify that $g(1)=0$

1 2) From the table of variations of the function g below :

x	0 $\qquad\qquad$ $+\infty$
$g'(x)$	$+$
$g(x)$	$-\infty$ ↗ $+\infty$

Show that $g(x)\le 0$ for every x in the interval $]0,1]$

and that $g(x)\ge 0$ for every x in the interval $[1,+\infty[$

II-We consider the numerical function f defined on the interval $]0,+\infty[$ by: $f(x)=x+\left(1-\frac{2}{x}\right)\ln x$

Let (C) be the curve of f in an orthonormal coordinate system $(O,\vec{i},\vec{j})$ (unit : $1cm$)

0.5 1) Show that $\lim\limits_{\substack{x\to 0\\ x>0}} f(x)=+\infty$ and interpret geometrically the obtained result.

0.25 2) a) Show that $\lim\limits_{x\to+\infty} f(x)=+\infty$

0.75 b) Show that the line (D) of equation $y=x$ is an asymptotic direction of the curve (C) at $+\infty$

1 3) a) Show that $f'(x)=\dfrac{g(x)}{x^2}$ for every x in the interval $]0,+\infty[$

075 b) Show that f is decreasing on the interval $]0\,,1]$ and increasing on the interval $[1\,,+\infty[$

0.25 c) Set up the table of variations of the function f on the interval $]0,+\infty[$

0.5 4) a) Solve in the interval $]0,+\infty[$ the equation $\left(1-\dfrac{2}{x}\right)\ln x=0$

0.5 b) Deduce that the curve (C) intersects the line (D) at two points, which the coordinates must be determined.

0.75 c) Show that $f(x)\le x$ for every x in the interval $[1\ ,\ 2]$ and then deduce the relative position of the curve (C) and the line (D) on the interval $[1\ ,\ 2]$

1 5) Sketch, the line (D) and the curve (C) in the same system coordinate $(O,\vec{i},\vec{j})$

(We admit that the curve (C) has a unique inflection point wich abscissa is between $2,4$ and $2,5$)

0.5 6) a) Show that $\displaystyle\int_1^2 \frac{\ln x}{x}dx=\frac{1}{2}(\ln 2)^2$

0.25 b) Show that the function $H: x\mapsto 2\ln x - x$ is a primitive of the function $h: x\mapsto \dfrac{2}{x}-1$ on the interval $]0,+\infty[$

0.5 c) Using an integration by parts, show that $\displaystyle\int_1^2\left(\frac{2}{x}-1\right)\ln x\,dx=(1-\ln 2)^2$

0.5 d) Calculate, in cm^2, the area enclosed between the curve (C), the line (D) and the lines of equations $x=1$ and $x=2$

III- We consider the numerical sequence (u_n) defined by :

$u_0=\sqrt{3}$ and $u_{n+1}=f(u_n)$ for every natural number n

0.5 1) Show by induction that $1\le u_n\le 2$ for every natural number n

0.5 2) Show that the sequence (u_n) is decreasing (you can use the result of the question II-4) c))

0.75 3) Deduce that the sequence (u_n) is convergent and determine its limit.