المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي
المركز الوطني للتقويم والامتحانات

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المسالك الدولية

الدورة الاستدراكية 2020

- الموضوع –

SSSSSSSSSSSSSSSSSSSS | RS 22E

المادة	الرياضيات	مدة الإنجاز	3
الشعبة أو المسلك	شعبة العلوم التجريبية مسلك علوم الحياة والأرض ومسلك العلوم الفيزيائية (خيار إنجليزية)	المعامل	7

GENERAL INSTRUCTIONS

- ✓ **The use of non- programmable calculator is allowed ;**
- ✓ **The exercises can be treated in the preferred order by the candidate ;**
- ✓ **The use of red color when writing solutions is to be avoided.**

COMPONENTS OF THE EXAM

✓ **The exam consists of three exercises and a problem , independent of each other according to the fields as follows :**

Exercise 1	**numerical sequences**	**2 points**
Exercise 2	**Complex numbers**	**5 points**
Exercise 3	**Study of numerical function and Calculating integrals**	**4 points**
Problem	**Study of numerical function, and numerical sequences**	**9 points**

- ✓ **ln denotes the Napierian logarithm function**
- ✓ $\overline{Z}$ **denotes the conjugate of the complex number** Z **and** $|Z|$ **it's module**

	Exercise 1 : (2 points)
	Consider the numerical sequence (u_n) **defined by** $u_0 = 1$ **and** $u_{n+1} = \dfrac{3u_n - 8}{2u_n - 5}$ **for every natural number** n
0.5	**1) Show that** $u_n < 2$ **for every natural number** n
	2) Consider $v_n = \dfrac{u_n - 3}{u_n - 2}$ **for every natural number** n
0.5	**a) Show that** (v_n) **is an arithmetical sequence of reason 2**
0.75	**b) write** v_n **in terms of** n **then deduce** u_n **in terms of** n
0.25	**c) calculate the limit of the sequence** (u_n)
	Exercise 2 : (5 points)
0.75	**1) Solve in the set of complex numbers** $\mathbb{C}$ **the equation :** $z^2 - \sqrt{2}z + 1 = 0$
	2) Let $a = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i$
0.75	**a) Write the number** a **in the trigonometrical form and deduce that** a^{2020} **is a real number**
0.5	**b) Let the complex number** $b = \cos\dfrac{\pi}{8} + i\sin\dfrac{\pi}{8}$**. Prove that** $b^2 = a$
	3) In the complex plane referred to an orthonormal direct coordinate system $(O, \vec{u}, \vec{v})$**, We consider the points** A**,** B **and** C **of respective affixes** a**,** b **and** c **such** $c = 1$**. Let** R **be the rotation with center** O **and angle** $\dfrac{\pi}{8}$**. The point** M' **of affix** z' **is the image of the point** M **of affix** z **by the rotation** R**.**
0.25	**a) Verify that** $z' = bz$
0.5	**b) Determine the image of the point** C **by the rotation** R**, and show that** A **is the image of the point** B **by** R**.**
0.75	**4) a) Show that** $\lvert a - b \rvert = \lvert b - c \rvert$ **and deduce the nature of the triangle** ABC
0.5	**b) Determine a measure of the oriented angle** $(\overrightarrow{BA}, \overrightarrow{BC})$
	5) Consider T the translation with vector $\vec{u}$ **, and let** D **be the image of the point** A **by T.**
0.25	**a) Verify that the affix of the point** D **is** $b^2 + 1$
0.75	**b) Show that** $\dfrac{b^2 + 1}{b} = b + \bar{b}$**, therefore deduce that the points** O**,** B **and** D **are colinear.**

	Exercise 3 : (4 points)
	Let u **be the numerical function defined on** $\mathbb{R}$ **by** $u(x)=e^{x}-2x+2-3e^{-x}$
0.5	**1) a) Show that for every** x **on** $\mathbb{R}$: $u'(x)=\dfrac{(e^{x}-1)^{2}+2}{e^{x}}$
0.25	**b) Set up the table of variations of** u **(the calculus of limits are not required)**
0.5	**c) Deduce the sign of the function** u **on** $\mathbb{R}$ **(Notice that** $u(0)=0$ **)**
	2) Let v **be the numerical function defined on** $\mathbb{R}$ **by** $v(x)=e^{2x}-2xe^{x}+2e^{x}-3$
0.5	**a) verify that for every** x **on** $\mathbb{R}$ $v(x)=e^{x}u(x)$
0.5	**b) Deduce the sign of the function** v **on** $\mathbb{R}$
0.5	**3) a) Show that the function** W **defined by** $W(x)=\frac{1}{2}e^{2x}+(4-2x)e^{x}-3x$ **is a primitive of the function** v **on** $\mathbb{R}$
0.5	**b) calculate** $\int_{0}^{2}v(x)\,dx$
0.75	**c) Show that** $\dfrac{9}{2}$ **is the absolute minima of the function** W **on** $\mathbb{R}$
	Problem : (9 points)
	I. Let g **be the numerical function defined on** $]0,+\infty[$ **by :** $g(x)=e^{1-x}+\dfrac{1}{x}-2$
0.5	**1) Show that** $g'(x)<0,$ **for every** x **in** $]0,+\infty[$
0.5	**2) Deduce the table of sign of** $g(x)$ **on the interval** $]0,+\infty[$ **; (Notice that** $g(1)=0$ **)**
	II. Let f **be the numerical function defined on** $]0,+\infty[$ **by :** $f(x)=(1-x)e^{1-x}-x^{2}+5x-3-2\ln x$ **and** (C) **its representative curve in an orthonormal coordinate system** $(O,\vec{i},\vec{j})$ **(unit: 2** cm**)**
0.5	**1)Show that** $\lim\limits_{\substack{x\to 0\\ x>0}} f(x)=+\infty$ **, then interpret geometrically the result**
0.5	**2) a)Show that** $\lim\limits_{x\to+\infty} f(x)=-\infty$
0.75	**b) Show that** $\lim\limits_{x\to+\infty}\dfrac{f(x)}{x}=-\infty$ **, then interpret geometrically the result**
1	**3) a) Show that for all** x **in** $]0,+\infty[$ **,** $f'(x)=(x-2)g(x)$
0.75	**b) Show that the function** f **is decreasing on** $]0,1]$ **and on** $[2,+\infty[$ **and it's increasing on** $[1,2]$
0.25	**c) Set up the table of variations of the function** f **on** $]0,+\infty[$ **, (take** $f(2)\simeq 1,25$ **)**

0.5	**4) Knowing that $f(3)\approx 0,5$ and $f(4)\approx -1,9$ show that the equation $f(x)=0$ admits an unique solution on the interval $]3,4[$.**
1	**5) Sketch the curve (C) in the coordinate system $(O,\vec{i},\vec{j})$**
	III. Let $h(x)=f(x)-x$ for every x on the interval $[1,2]$
0.5	**1) a) Using the table of variations of the function h oposite, show that $f(x)\leq x$ for every x of the interval $[1,2]$**
0.25	**b) Show that 1 is the unique solution of the equation $f(x)=x$ on the interval $[1,2]$**
	2) Let (u_n) be the numerical sequence defined by: $u_0=2$ and $u_{n+1}=f(u_n)$ for every n on $I\!N$
0.75	**a) Show by induction that $1\leq u_n\leq 2$ for every n on $I\!N$**
0.5	**b) Show that the sequence (u_n) is decreasing.**
0.75	**c) Deduce that the sequence (u_n) is convergent and calculate $\lim_{n\to+\infty} u_n$.**

x	1	2
$h(x)$	0 ↘	$h(2)$